Chapter 2 Lecture Notes: Economics for MBAs and Masters of Finance

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In this chapter, we define the theory of firms and growth.

We also map this theory to available data.

Although we present a naive view about how firms operate, this view provides powerful predictions that appear to hold in the data, at least over long periods of time.

Question you must know how to answer:

1. Using 50 words or less, explain why the following example of Marx's labor theory of value is misleading:

> *A worker in a factory is given $30 worth of material, and after working 3 hours producing a good, and using $10 worth of fuel to run a machine, he creates a product which is sold for $100. According the Marx, the labor and only the labor of the worker increased the value of the natural materials to $100. The worker is thus justly entitled to a $60 payment, or $20 per hour.*

Suppose output Y is produced using three inputs:

- technology z
- capital K
- labor L

Suppose also that capital and labor are homogeneous, and all firms have access to the same technology.

We do this not because we think it true, but because these simplifications lead to tractability.

Cobb Douglas Production

Given the inputs, what would a production function for Y look like?

Economists think output is produced according to a "Cobb-Douglas" production function.

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha} \quad (1)$$

where $0 < \alpha < 1$ and $0 < 1 - \alpha < 1$.

α is commonly called the "capital share" of production.

This production function has two important properties

- Constant returns to scale
- Declining marginal products

Some quick math:

$$X^n * X^m = X^{n+m}$$

$$\begin{aligned} 2^3 * 2^4 &= 2^7 \\ 2^{0.3} * 2^{0.7} &= 2^{1.0} \end{aligned}$$

Constant Returns to Scale

Constant returns to scale says the following: If you double both the capital stock and labor input, output doubles.

$$\begin{aligned} Y_t &= z_t * (2 * K_t)^{\alpha} (2 * L_t)^{1-\alpha} \\ Y_t &= z_t * 2^{\alpha} * K_t^{\alpha} * 2^{1-\alpha} * L_t^{1-\alpha} \\ Y_t &= z_t * 2^{\alpha} * 2^{1-\alpha} * K_t^{\alpha} * L_t^{1-\alpha} \\ Y_t &= 2 * z_t * K_t^{\alpha} * L_t^{1-\alpha} \\ Y_t &= 2 * \left[z_t K_t^{\alpha} L_t^{1-\alpha} \right] \end{aligned}$$

Therefore, output of N small firms is the same as one firm that is N times as large.

When the assumption of perfect competition is married to constant-returns to scale production, we can pretend as if there is only one "representative" firm in the economy that is a "price taker."

This means our one representative firm in the economy acts as if the prices of its inputs – labor and capital – are outside of its control

- You will likely think this is true for capital.
  Walmart cannot dictate its stock price.
- Is it true for labor? Perhaps.

Some more quick math:

1. The derivative of a function $y = f(x)$ describes how much y would increase if x were to increase by one unit.
2. The derivative of $y = A * X^n$ with respect to X is $n * A * X^{n-1}$.

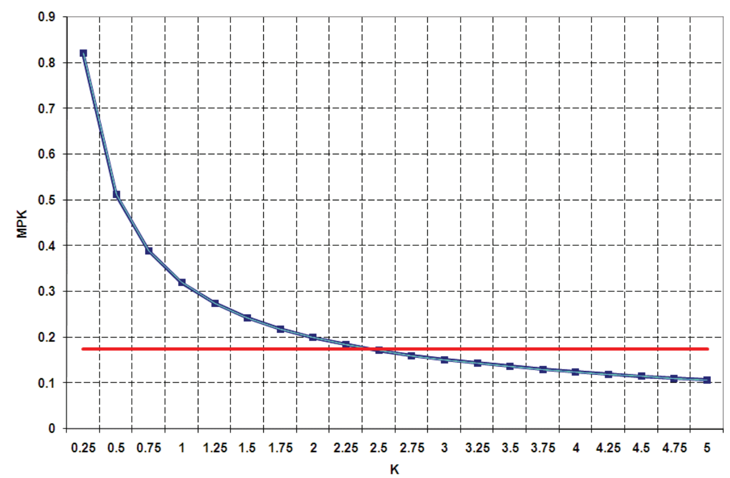
Declining Marginal Products

The marginal product of capital (MPK) is the derivative of Y_t with respect to K_t

$$\text{MPK} = \frac{\partial Y_t}{\partial K_t} = \frac{\partial\left[z_t K_t^{\alpha} L_t^{1-\alpha}\right]}{\partial K_t} = \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} = \alpha z_t \left(\frac{L_t}{K_t}\right)^{1-\alpha}$$

Since $1 - \alpha > 0$, the MPK is decreasing as K_t increases, all else held fixed.

Figure: Example of MPK, with $\alpha = 0.32$, $z = 1$ and $L = 1$

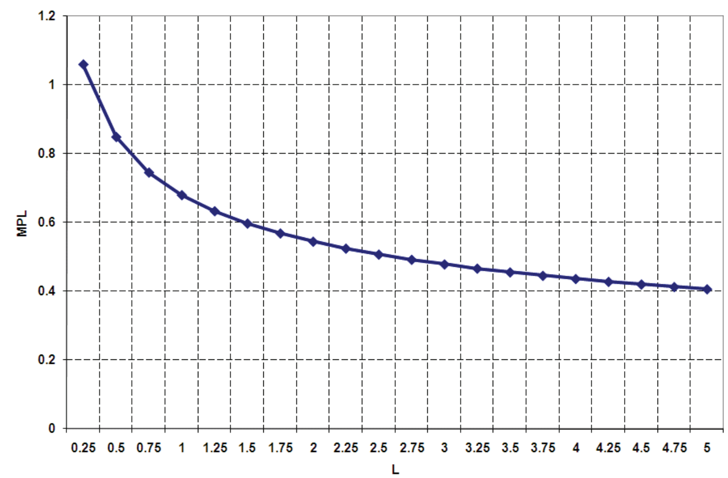


The marginal product of labor (MPL) is the derivative of Y_t with respect to L_t

$$\begin{aligned} \text{MPL} = \frac{\partial Y_t}{\partial L_t} &= \frac{\partial \left[z_t K_t^{\alpha} L_t^{1-\alpha} \right]}{\partial L_t} \\ &= (1-\alpha)\, z_t K_t^{\alpha} L_t^{-\alpha} \\ &= (1-\alpha)\, z_t \left(\frac{K_t}{L_t} \right)^{\alpha} \end{aligned}$$

Since $\alpha > 0$, the MPL is decreasing as L_t increases, all else held fixed.

Figure: Example of MPL, with $\alpha = 0.32$, $z = 1$ and $K = 1$



Profit Maximization

Profits are equal to the value of output less the cost of inputs.

Assume that there is no inflation: that is, fix all prices to 1.0.

- This will be the assumption for much of the rest of the book.
- This way, we will not confuse "nominals" with "reals": All variables in our models from now on will be in reals.

Then the firm maximizes profits by choosing K_t and L_t to maximize

$$\underbrace{z_t K_t^{\alpha} L_t^{1-\alpha}}_{\substack{\text{value of} \\ \text{output}}} - \underbrace{r_t * K_t}_{\substack{\text{expenditures} \\ \text{on capital}}} - \underbrace{w_t * L_t}_{\substack{\text{expenditures} \\ \text{on labor}}}$$

- r_t is the rental rate on one unit of capital,
- w_t is the wage rate on one unit of labor,
- since z_t has no price, we have assumed technology is freely available to all.

How does the firm maximize profits? Easy!

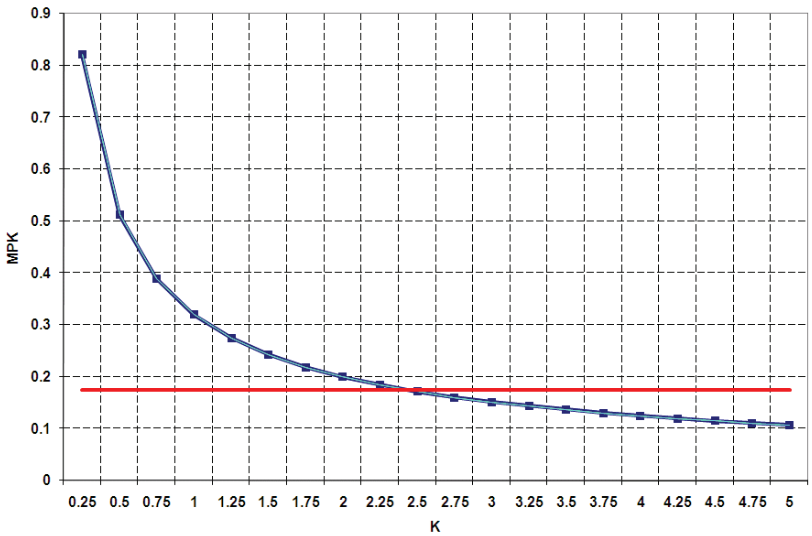
- Firm chooses *capital* until the marginal revenue associated with an additional unit of capital (MPK) is equal to the marginal cost of an additional unit of capital.
  $MPK = r$
- Firm chooses *labor* until the marginal revenue associated with an additional unit of labor is equal to the marginal cost of an additional unit of labor.
  $MPL = w$

Fixing ideas: Suppose that

- The cost of one additional unit of capital is \$0.175.
- The extra revenue from one more unit of capital is \$0.23.
- What to do? Get more capital!

The optimal amount of capital employed by the firm is bounded because the marginal product of capital is declining.

Figure: Example of MPK $= r$, with $r = 0.175$



When we apply these ideas to the math – Cobb-Douglas production function – we get a few special relationships.

Optimal Capital

To find the maximum, take derivative with respect to K_t:

$$\begin{aligned}\alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} &= r_t \\ \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} * K_t &= r_t * K_t \\ \alpha Y_t &= r_t * K_t\end{aligned}$$

- Line 1: The MPK is set to the rental rate for capital.
- Line 3: Expenditures on capital services are a constant fraction (α) of the value of output.
  - Every dollar spent on capital services is a dollar earned (by someone) as capital income.
  - Data: capital income accounts for about 32 percent of all income (last chapter). Set $\alpha = 0.32$.

Optimal Labor

To find the maximum, take derivative with respect to L_t:

$$\begin{aligned}(1-\alpha) z_t K_t^{\alpha} L_t^{-\alpha} &= w_t \\ (1-\alpha) Y_t &= w_t * L_t\end{aligned}$$

- Line 1: The MPL is the pre-tax wage rate.
- Line 2: Expenditures on labor are a constant fraction $(1-\alpha)$ of the value of output.
  - An estimate of the "labor share" of production is therefore 68 percent.

Optimal Profits

Note that our framework has implied that optimal profits are zero.

You might have guessed this would be the case given we have assumed that there are lots of small firms in the economy acting as price-takers.

$$\begin{aligned}r_t K_t &= \alpha Y_t \\ w_t L_t &= (1-\alpha) Y_t \\ \text{Implies } r_t K_t + w_t L_t &= Y_t\end{aligned}$$

- Of course, firms make *accounting* profits.
- Accounting profits are payments to capital!

So, what is the answer?

1. Using 50 words or less, explain why the following example of Marx's labor theory of value is misleading:

> *A worker in a factory is given $30 worth of material, and after working 3 hours producing a good, and using $10 worth of fuel to run a machine, he creates a product which is sold for $100. According the Marx, the labor and only the labor of the worker increased the value of the natural materials to $100. The worker is thus justly entitled to a $60 payment, or $20 per hour.*

Growth Accounting

Productivity, or output per hour, is

$$\frac{Y_t}{L_t}$$

which is also called the "average product of labor."

Note that according to our rules for optimal firm behavior

$$(1-\alpha)\frac{Y_t}{L_t} = w_t.$$

Wages increase when productivity increases. Productivity is therefore *key*.

What could cause productivity to rise? That is, what increases output holding the labor input constant?

Recall our production function:

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}$$

So, for Y_t to increase holding L_t fixed, either z_t or K_t must increase.

To understand the sources of growth, consider output in periods t and $t+1$:

$$\begin{aligned} Y_{t+1} &= z_{t+1} \quad K_{t+1}^{\alpha} \quad L_{t+1}^{1-\alpha} \\ Y_t &= z_t \quad K_t^{\alpha} \quad L_t^{1-\alpha} \end{aligned}$$

Divide by the population N_t (and recall $N_t = N_t^{\alpha} N_t^{1-\alpha}$).

$$\begin{aligned} \frac{Y_{t+1}}{N_{t+1}} &= z_{t+1} \quad \left(\frac{K_{t+1}}{N_{t+1}}\right)^{\alpha} \quad \left(\frac{L_{t+1}}{N_{t+1}}\right)^{1-\alpha} \\ \frac{Y_t}{N_t} &= z_t \quad \left(\frac{K_t}{N_t}\right)^{\alpha} \quad \left(\frac{L_t}{N_t}\right)^{1-\alpha} \end{aligned}$$

Denote lower case variables y, k, and l as per-capita variables:

$$\begin{aligned} y_{t+1} &= z_{t+1} \quad k_{t+1}^{\alpha} \quad l_{t+1}^{1-\alpha} \\ y_t &= z_t \quad k_t^{\alpha} \quad l_t^{1-\alpha} \\ \Rightarrow \frac{y_{t+1}}{y_t} &= \frac{z_{t+1}}{z_t} \quad \left(\frac{k_{t+1}}{k_t}\right)^{\alpha} \quad \left(\frac{l_{t+1}}{l_t}\right)^{1-\alpha} \end{aligned}$$

Take logs and denote g_y, g_z, g_k, and g_l as growth rates

$$\ln(1+g_y) = \ln(1+g_z) + \alpha \ln(1+g_k) + (1-\alpha)\ln(1+g_l)$$

Use log approximation:

$$\Rightarrow g_y \approx g_z + \alpha g_k + (1-\alpha) g_l$$

$$\Rightarrow g_y \approx g_z + \alpha g_k + (1-\alpha) g_l$$

Growth in per-capita output comes from either:

- Growth in technology: g_z
- Growth in the real per-capita stock of capital: αg_k
- Growth in the per-capita labor input: $(1-\alpha) g_l$

Growth in Developed Countries

- Since the 1950s and perhaps earlier, fraction of total discretionary time spent working on a per-capita basis has been roughly constant (will show this later)
- Total time is bounded: Only 24 hrs in a day
- Real GDP cannot sustainably increase through growth in l. Set $g_l = 0$. (To be discussed in more detail in a few slides).
- Implies for a developed economy

$$g_y \approx g_z + \alpha g_k$$

Suppose that technology is not increasing, so $g_z = 0.0$.

Since $g_z = 0$, $g_y \approx \alpha * g_k$.

$\alpha < 1$ implies that per-capita output is increasing less rapidly than per-capita capital.

To see this, consider the following table of examples

g_k	$g_y = 0.32 * g_k$
0.10	0.032
0.05	0.016
0.01	0.003

Now recall that the required rental rate on capital is

$$\alpha\left(\frac{Y_t}{K_t}\right) = \alpha\left(\frac{Y_t/L_t}{K_t/L_t}\right) = \alpha\left(\frac{y_t}{k_t}\right) = r_t$$

If output were to increase solely through the accumulation of capital, the output to capital ratio would fall.

Eventually, this would drive down the marginal product of capital to less than the required rental rate on capital.

Conclusion: Capital accumulation (alone) can not sustainably increase productivity. Technology must increase for sustained growth in GDP!

Balanced Growth

Appears that the U.S. has been on a Balanced Growth path

- Real interest rates (the pre-tax pre-depreciation marginal product of capital) are trendless,
- The per-capita labor input is trendless,
- Output, consumption, investment, and capital all increase at the same rate,
- The rate of growth of output, consumption, investment, and capital is intrinsically linked to the rate of growth of technology.

With $g_l = 0$, in a developed economy

$$g_k = g_z + \alpha * g_k$$

With balanced growth and trendless interest rates, $g_k = g_y$. This implies

$$g_y = g_k = \frac{g_z}{1-\alpha}.$$

Now consider GDP accounting, but for simplicity ingore government spending and net exports:

$$Y_t = C_t + I_t \ .$$

Use the relationship that $K_{t+1} = K_t (1-\delta) + I_t$.

$$Y_t = C_t + K_{t+1} - K_t (1-\delta) \ .$$

Divide by Y_t and use trick that $1/Y_t = (1/Y_{t+1})(Y_{t+1}/Y_t)$.

$$1 = \frac{C_t}{Y_t} + \left(\frac{K_{t+1}}{Y_{t+1}}\right)\left(\frac{Y_{t+1}}{Y_t}\right) - \left(\frac{K_t}{Y_t}\right)(1-\delta) \ .$$

$$1 = \frac{C_t}{Y_t} + \left(\frac{K_{t+1}}{Y_{t+1}}\right)\left(\frac{Y_{t+1}}{Y_t}\right) - \left(\frac{K_t}{Y_t}\right)(1-\delta) \ .$$

Along a balanced growth path:

- K_t/Y_t and K_{t+1}/Y_{t+1} are constant.
- Y_{t+1}/Y_t is constant: $g_z/(1-\alpha)$.
- δ is assumed constant.
- Since 1 is a constant, C_t/Y_t must also be constant.
- Since $1 = C_t/Y_t + I_t/Y_t$, this means I_t/Y_t is also constant.

Growth in Developing Countries

We have shown that a developed economy needs growth in technology for sustained growth in per-capita output.

What explains growth in an immature economy, such as China? In China

- The labor input L_t is increasing as inefficient farm work is moving to efficient factory work. $g_l > 0$.
- The technology input z_t is increasing as Chinese firms allow outside experts to design factories and processes. $g_z > 0$.
- The capital input K_t is increasing (before 1990, China had very little in the way of capital). $g_k > 0$.

So, the growth rate of Chinese GDP is high because all three inputs are growing rapidly. *Eventually*, Chinese GDP will increase at the same rate as technology, just as in the U.S.

Real Per-Capita (PC) GDP (constant US $2000) in 1973 and 2003, and Growth in Real PC GDP from 1973-2003, 10 Poorest Countries as of 1973.

Country	Real PC GDP 1973	Real PC GDP 2003	Growth in Real PC GDP 1973-2003
Bhutan	$250	$934	4.5%
Ethiopia	$503	$688	1.0%
North Korea	$542	$1,429	3.3%
China	$561	$4,970	7.5%
Tanzania	$572	$912	1.6%
Malawi	$593	$771	0.9%
Guinea-Bissau	$631	$584	-0.3%
Mali	$638	$1,184	2.1%
Burkina Faso	$692	$1,071	1.5%
Cambodia	$763	$580	-0.9%
Average, Bottom 10 (1973)	$574	$1,312	2.1%

Real Per-Capita (PC) GDP (constant US $2000) in 1973 and 2003, and Growth in Real PC GDP from 1973-2003, 10 Richest Countries as of 1973.

Country	Real PC GDP 1973	Real PC GDP 2003	Growth in Real PC GDP 1973-2003
Germany	$15,218	$25,188	1.7%
Australia	$15,944	$27,872	1.9%
New Zealand	$15,947	$22,195	1.1%
Canada	$16,034	$27,845	1.9%
Netherlands	$16,294	$26,157	1.6%
Sweden	$16,470	$26,136	1.6%
Denmark	$18,126	$27,970	1.5%
Luxembourg	$19,305	$49,262	3.2%
United States	$19,552	$34,875	1.9%
Switzerland	$23,074	$28,792	0.7%
Average, Top 10 (1973)	$17,596	$29,629	1.7%

Barriers to Growth

- China's experience of fast growth is atypical.
- Most countries do not "catch up." This is a puzzle.
- One reason: The effective tax rate on capital income may be very high!
- This is an important lesson for you. The capital-income tax is *not* just a tax on the rich (who own all the capital). It is a tax on all workers as well.
- Why? According to our production function, workers need capital to be productive.

The key insight is that taxes draw a wedge between the rate of return on capital earned by households, denoted $\widehat{r}_t$, and the marginal product of capital at the firm level r_t.

$$\widehat{r}_t = (1 - \tau_k)(r_t - \delta)$$

Example:

- You loan $100 of computer equipment to a friend for a year.
- The friend pays you $17.50. This is r_t.
- However, the equipment depreciates during the year by $\delta = $ 5-1/2 percent – it is only worth $94.50 once it is returned to you.
- So your pretax income is $17.50 - $5.5 = $12.00.
- If the capital-income tax is $\tau_k = 50$ percent, you keep 0.50*$12.00 = $6. This is $\widehat{r}_t$.

Now fix the labor input (again) to $L_t = 1.0$ and then recall that

$$r_t = \alpha K_t^{\alpha - 1}$$

This implies

$$K_t = \left[\frac{1}{\alpha}\left(\frac{\widehat{r}_t}{1 - \tau_K} + \delta\right)\right]^{\frac{1}{\alpha - 1}}$$

Example with $\widehat{r}_t = 0.06$.

τ_K	K_t	Y_t	w_t
0.40	2.904	1.407	0.956
0.60	1.925	1.233	0.839
gap	51%	14%	14%

Measurement: Capital

How are the inputs to production K_t, L_t, and z_t measured?

Data on the capital stock is collected by the BEA, and is available in a set of tables that are complementary to the NIPA, the *Fixed Asset Tables*.

Bureau of Economic Analysis
Fixed Asset Table

Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods
[Billions of dollars; yearend estimates]
Today is: 6/24/2008 Last Revised on August 08, 2007

Line		2006
1	**Fixed assets and consumer durable goods**	**44,432.0**
2	**Fixed assets**	**40,556.9**
3	Private	31,818.5
4	Nonresidential	14,715.0
5	Equipment and software	5,027.9
6	Structures	9,687.1
7	Residential	17,103.5
8	Government	8,738.5
9	Nonresidential	8,397.1
10	Equipment and software	856.6
11	Structures	7,540.6
12	Residential	341.4
13	**Consumer durable goods**	**3,875.1**
	Addenda:	
14	**Private and government fixed assets**	**40,556.9**
15	Nonresidential	23,112.1
16	Equipment and software	5,884.4
17	Structures	17,227.6
18	Residential	17,444.9
19	**Government fixed assets**	**8,738.5**
20	Federal	1,829.0
21	State and local	6,909.4

The way the BEA computes the capital stock is via "Perpetual Inventory Accounting." That is, given a period t estimate of the capital stock, it computes the period $t+1$ estimate using the formula

$$K_{t+1} = K_t - \delta K_t + I_t$$

where I_t is investment as measured in the NIPA and δ is the depreciation rate on capital.

The depreciation rates are, generally speaking, derived from academic studies – but of course that does not automatically mean they are correct.

This method requires an initial estimate of the capital stock.

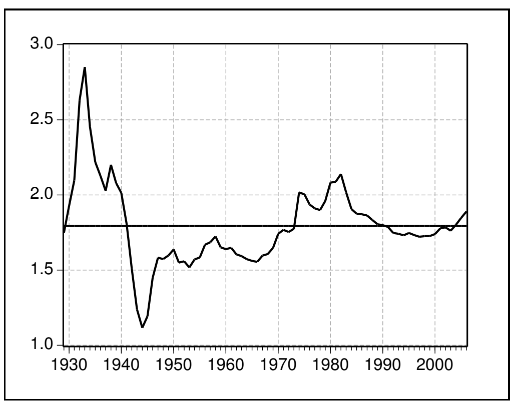
So, what is the Capital-Output ratio? Let's assume residential structures are not used to produce GDP, except for rents on housing, which is the consumption of housing services, but all other non-defense capital is used:

Line	Item	Nominal Value (2006)
3	Private fixed assets (total)	\$31,818.5
21	+ State and local assets	\$6,909.4
NA	+ Federal Non-defense assets	\$708.7
11	- Residential Fixed Assets	-\$17,103.5
	Total Assets Used for GDP	\$22,333.1

Nominal GDP (\$13,194.7) less the nominal consumption of housing services (\$1,381.3) in 2006 is \$11,813.4.

So the capital-output ratio in 2006 is 1.89.

Figure: The Ratio of the Nominal Value of Capital to Nominal Annual Output, 1929-2006



Recall that when firms profit maximize and output is produced using a Cobb-Douglas production function, then

$$r_t = \alpha \left(\frac{Y_t}{K_t} \right)$$

Suppose that the capital-output ratio is, on average, 1.8. Then the average output-capital ratio is 0.556.

Recall we have estimated $\alpha = 0.32$ (capital's share of income).

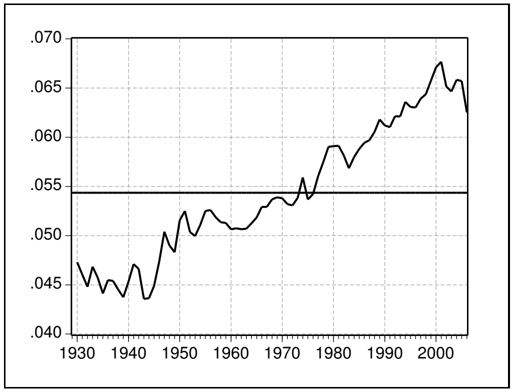
This means that the economy-wide rental rate on capital is $r_t = 0.32 * 0.556 = 0.178$. Keep this in the back of your mind.

Next, let's use more data on depreciation, also in the Fixed Assets tables, to estimate the economy-wide depreciation rate on capital.

The depreciation rate can be calculated as, approximately, the value of depreciation in year t divided by the stock of capital in year $t-1$.

Yearly estimates of the economy-wide depreciation rate for the aggregate stock of capital is shown on the next page.

Figure: The Depreciation Rate of Capital, δ, 1930-2006



The depreciation rate has averaged 5.4% per year, but has been increasing and was 6.3% per year in 2006.

Let's assume that the required after-tax rate of return on capital is 6 percent. Then, we can solve for the historical-average (1929-2007) implicit tax rate on capital in the United States

$$\begin{aligned} \widehat{r}_t &= (1-\tau_k) * (r_t - \delta) \\ 0.06 &= (1-\tau_k) * (0.178 - 0.054) \\ \rightarrow \tau_k &= 0.515 \end{aligned}$$

This is higher than a standard estimate of the tax rate on capital income in the US, about 40 percent.

Measurement: Labor

Economists use two surveys to measure how many hours people are working (in the marketplace). Both surveys are collected by the U.S. Dept of Labor, Bureau of Labor Statistics (BLS):

- Monthly payroll survey
  Non-random sample. Survey of hours at 390 thousand big firms employing 47 million.
- Monthly household survey
  Random sample of 50 thousand households in 792 sample areas.

Most economists view the payroll survey as a more accurate picture of how many hours are spent working in the aggregate.

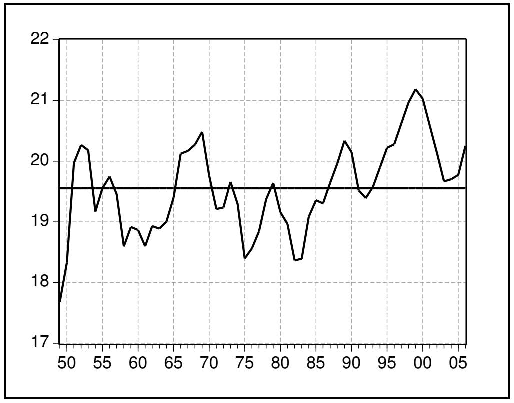
Note that, like capital, we do not "quality adjust" the hours. We might get a different picture of the cyclicality of labor if we were to quality adjust hours.

But anyway, the quality-unadjusted hours series is readily available.

The hours data suggest that the average hours worked per-week, per-person, is trendless.

However, hours worked is volatile around its trend

Figure: Per-Capita Hours Worked per Week, 1949-2006



This is hours worked per week divided by the civilian non-institutionalized population age 16 and older. *Question:* Why is it only 19.5?

We use two concepts to describe deviation of labor hours around its trend. Neither concept is perfect.

- **Labor force participation rate**
  Labor Force divided by "Number of Potential Workers." The participation rate, on average in 2007, was 66 percent. NOTE that the number of potential workers *excludes* home-makers. This seems odd if you believe that home-workers can switch from home-work to market-work (for the right wage).[1]
- **Unemployment rate**
  Percentage of individuals in the labor force that are unemployed and actively looking for a job. The unemployment rate, on average in 2007, was 4.6 percent.

[1] Alternatively, home-makers could charge each other for child-care, education, laundry, etc. and then it would be measured as market work.

Business economists and bond traders pay close attention to the labor-market statistics:

- The labor market data are released more timely than estimates of GDP and
- Volatility in hours worked is about the same as volatility of GDP.

So the labor market data provide an initial snapshot of current quarter real GDP growth.

Some more quick math (natural logs):

1. $\ln(x * y) = \ln(x) + \ln(y)$
2. $\ln(x^n) = n * \ln(x)$

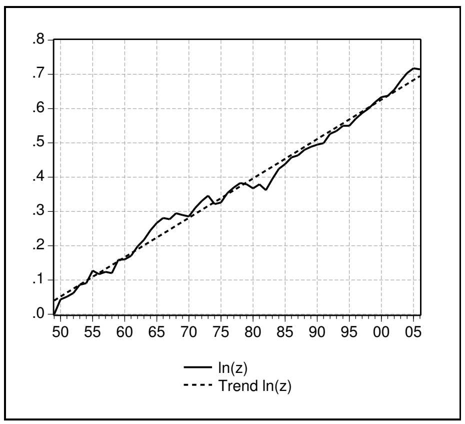
Measurement: Technology

We do not get a direct read on the level of technology. Rather, we infer what technology must be given estimates of output, capital, and labor:

$$\begin{aligned} Y_t &= z_t K_t^{\alpha} L_t^{1-\alpha} \\ \ln(Y_t) &= \ln(z_t) + \alpha \ln(K_t) + (1-\alpha)\ln(L_t) \\ \rightarrow \ln(z_t) &= \ln(Y_t) - \alpha \ln(K_t) - (1-\alpha)\ln(L_t) \end{aligned}$$

So, with an estimate of α, and data on real output, the real stock of capital, and hours worked, we can infer the level of technology.

Figure: $\ln(z_t)$ and its Trend, with $\ln(z_t)$ Rescaled to 0.0 in 1949, 1949-2006



z_t has been increasing at about 1.1 percent per year since 1949.